

## EuPhO-2019: Solution of the experimental problem.

### Task 1. Sensitivity of receiver.

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### Task 2. Wavelength in water.

One possible setup is shown in problem text. Measurements are made by placing receiver on top of box lid. Such configuration allows to measure interference pattern created from two semi-transparent surfaces (air-water interfaces as well as reflection from aluminium foil). By adding water in stepwise manner and thus increasing layer height the phase difference between direct and reflected wave is changed giving water layer height difference  $\Delta h = \lambda/2$  between adjacent maxima or minima. Experimental data (shown in Figure 1) yields wavelength  $\lambda \approx 4.5 \text{ cm}$ . Plausible range of values can fall within (4–5 cm) Here we did not start measurements from zero thickness layer (water already covered the bottom of the box since initial results might fluctuate intensively). It is important to reduce any holes and creases in foil by flattening it and pressing it closer to plastic bottom. By not doing so the experimenter is most likely to introduce overpowering background noise to the results thus eliminating the possibility of obtaining any meaningful data. Escaped radiowaves creates complex reflection and interference pattern bypassing water layer through air without significant power loss. A good indication for a well made setup is  $8 \text{ dB}$  or higher difference in measured power between adjacent maximum and minimum.

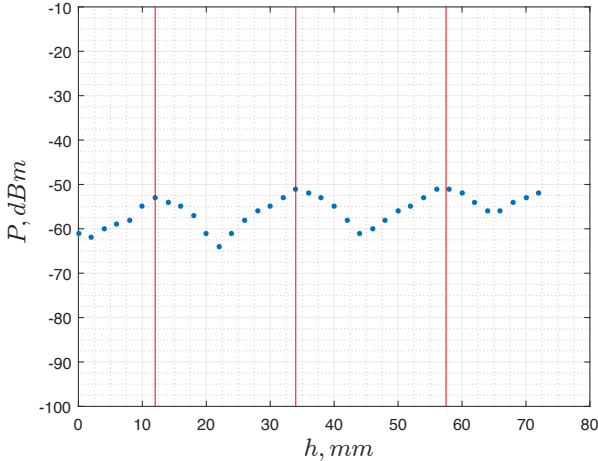


Figure 1: Interference pattern.

### Task 3. Attenuation in water.

In general the electric field vector of propagating wave in metallic tube filled with medium can be written in form

$$\vec{E} = \vec{E}_0(r, \varphi) e^{-\alpha z} e^{i(kz - \omega t)} \quad (1)$$

where in the case of water  $\alpha > 0$ . By solving Maxwell's equations it leads to dispersion relation

$$\omega^2 = (k_*^2 + k^2) c^2 \quad (2)$$

Water has high refraction coefficient for radio waves in emitters frequency range resulting in significantly smaller wavelength inside water leading to condition  $\omega \geq k_* c$ , where  $c$  stands for speed of light in the medium filling the waveguide.

Here as well are two possible setups available. The obvious one is to carefully wrap the tube inside foil. There can't be any holes. In order to test how well the tube is sealed one can insert emitter at the bottom within tube to find out that all signal has diminished. Since the wavelength is much shorter in water compared to air, tube acts as a waveguide. However for  $\alpha > 0$  there exists attenuation attributed exclusively to water which can be measured by dipping emitter inside water and measuring power of the waves. Keep in mind that if water is not filled up to the same level as foil wrapping, tube stops acting as a waveguide and therefore not all of decayed power can be attributed to water attenuation.

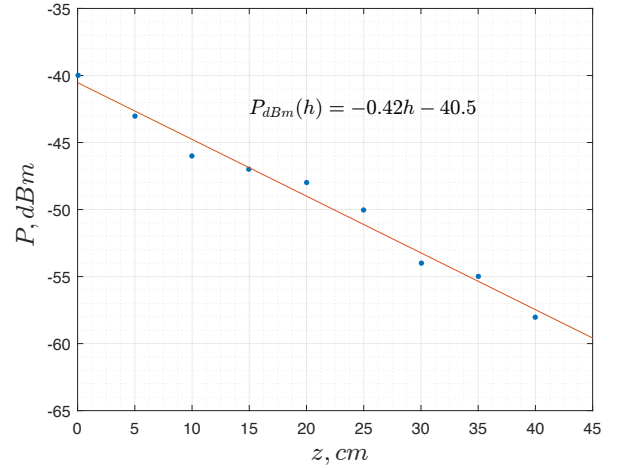


Figure 2: Water attenuation measurements.

This task is prone to many errors and fluctuations because relative orientation might change during immersion of emitter. To evade this problem it is necessary to repeat measurements, as well as consistently look for highest possible replicable value achievable for given depth. Data in Figure 2 gives an estimate of about  $0.4 \text{ dB/cm}$  or  $2 \text{ dB}$  per every  $5 \text{ cm}$ . Over  $5 \text{ cm}$  the attenuation factor is  $10^{0.2} \approx 1.6$ . Careful measurements should yield attenuation rate of  $0.35 - 0.5 \text{ dB/cm}$  or attenuation factor  $1.50 - 1.78$  per  $5 \text{ cm}$ .

### Task 4a. Decaying modes in air-filled waveguides

The most stable way to measure power of the waves received by the receiver is by placing tube horizontally and using ruler to push emitter inside tube. It can be done vertically using rope but results are expected to be a lot worse since single relative orientation is not maintained throughout experiment as well as reading is affected by stretching of rope.

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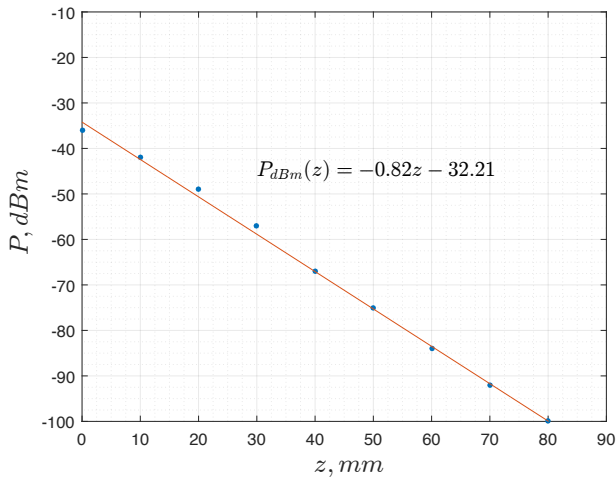


Figure 3: Power decay in 46mm tube.

Slope coefficient  $m$  determined from Figure 3 differs from  $\mu$  because of different units used. Time averaged electric field  $\langle E \rangle = E_0 e^{-\mu z}$ . Since wave power  $P \propto E^2$  this leads that  $\mu = 0.05 \cdot (\ln 10) \cdot m$ . In case of 46 mm tube, the decay rate  $\mu \approx 94.6 \text{ m}^{-1}$ . This part has to be done as precisely as possible. Otherwise it will almost certainly give imprecise results in later tasks. Good range for  $\mu$  extends between 93 and 96  $\text{m}^{-1}$ .

#### Task 4b.

Constant  $k_*$  is only dependent from diameter of the tube. Dimension analysis on dispersion relation tells us that  $k_* = A/d^2$  where  $A$  is dimensionless number. One can experimentally verify that waves are not propagating freely in none of the given tubes. That leads to imaginary  $ik = \mu$  and suggesting following functional dependence

$$\mu^2 = A/d^2 - k_0^2 \quad (3)$$

where  $k_0^2 = \omega^2/c^2$ . By plotting  $\mu^2(d^{-2})$  one should verify that dependence is linear in nature (Figure 4).

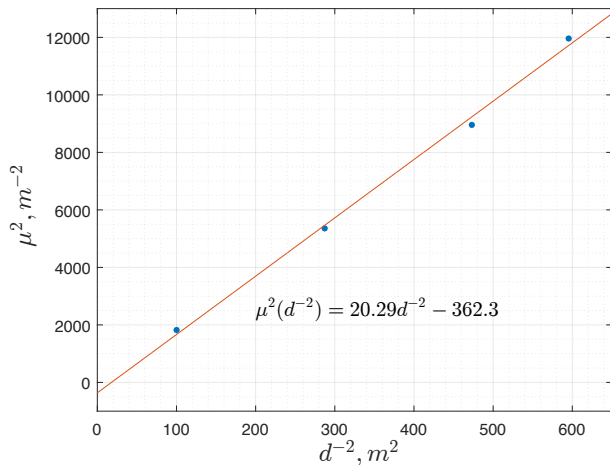


Figure 4: Functional dependence of decay rates and diameters

#### Task 5. Wavelength in air and refraction coefficient in water.

From Figure 4 one can find intercept with  $\mu^2$  axis (case of infinitely large diameter which corresponds to free space). According to equation (3), wavelength can be found  $-\mu^2 =$

$k_0^2 = \omega^2/c^2 = 4\pi^2/\lambda^2$ . Experimental data gives wavelength in air  $\lambda_0 \approx 33 \text{ cm}$  (actual wavelength is 34 cm). Since previous tasks has to be done really accurate, other values in range between 20cm and 40cm are good enough estimates for wavelength in air.

Now, finding the coefficient of refraction for water becomes trivial:  $n = \lambda_0/\lambda \approx 7.3$ . Results estimated are the result of previous tasks. Therefore accuracy is already predefined previously.

Part & Total	Criteria	Max Points	Score
<b>Task 1: 1.0</b>	Sketch showing a reasonable method as per solutions or otherwise acceptable	0.3	
	Lowest value $< -115$ dBm <i>OR</i> (below -130: 0 points)	0.5	
	Lowest value $< -105$ dBm	(0.2)	
	Correct conversion to mW	0.2	
<b>Task 2: 6.0</b>	Interferometer in water		
1.5	Diagram of setup, showing key points: sealing, positions of receiver and emitter, showing interference measurement	1.0	
	Equations for physics to measure $\lambda$ (eg $\Delta h = \lambda/2$ )	0.5	
1.0	Table of reasonable measured values:		
	0.1 For each 4 data points up to 24	0.6	
	For at least three maxima clearly present <i>OR</i> If two maxima clearly present	0.4 (0.2)	
1.5	Correctly plotted graph of data from table showing peaks and correct structure:		
	Correct labelling of axes and title	0.2	
	All data points plotted correctly (0.1 for each 3)	0.8	
	Positions of maxima / minima clearly marked and defined	0.5	
0.5	Quality: at least 8 dB between transmission maxima and minima	0.5	
1.5	Value for $4.0 \text{ cm} \leq \lambda \leq 5.0 \text{ cm}$ <i>OR</i>	1.5	
	Value for $3.5 \text{ cm} \leq \lambda \leq 5.5 \text{ cm}$	(0.7)	
<b>Task 3: 3.0</b>	Sketches of method, including full shielding with foil or Al tube, large depth of water	0.4	
	Showing shielding including bottom of pipe	0.3	
	Evidence of repeated measurements <i>or</i> careful checking for optimal orientation	0.2	
	Measurements span range of at least 35 cm <i>OR</i>	0.2	
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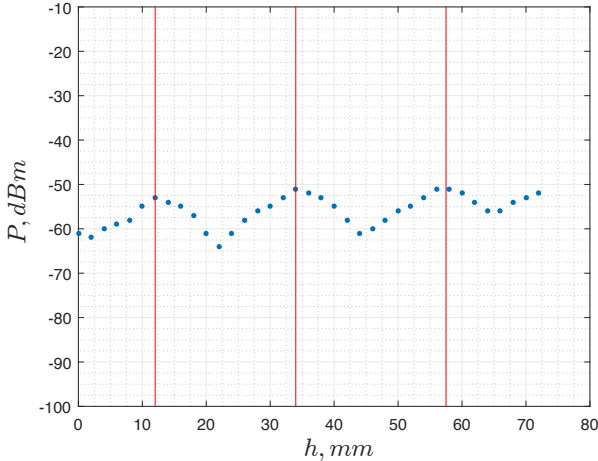


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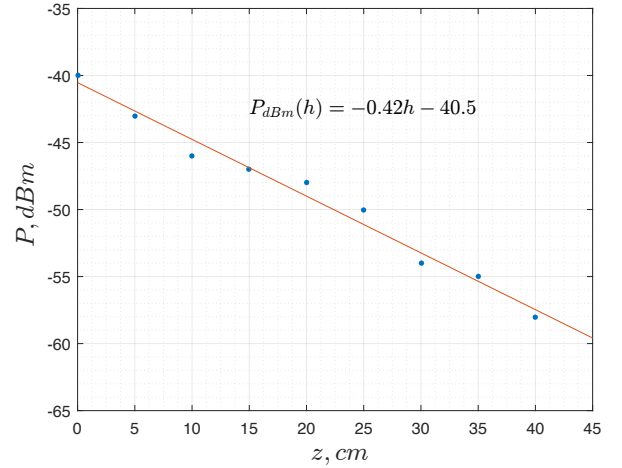


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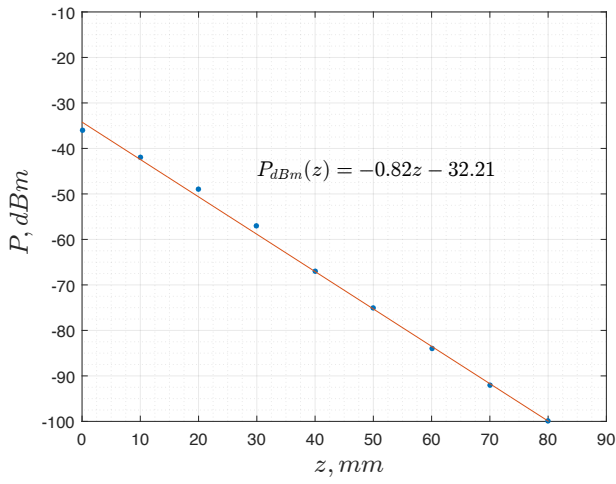


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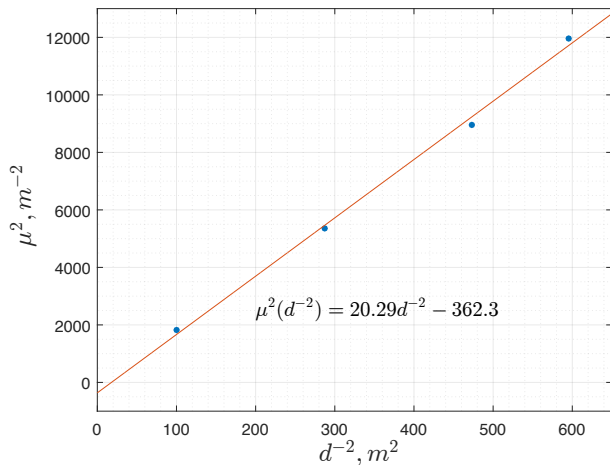


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<b>4b: 5.0</b>	Dependence of $\mu$ on $d$ Dimensional argument to get $k_* = A/d^2$ Correct functional dependence $\mu^2 = A/d^2 - k_0^2$ Choosing to plot $\mu^2$ vs $d^{-2}$	0.5 1.0 0.3	
1.6	Up to 8 sets of measurements for each tube (even if they appear in part a): 0.1 for each set of measurements with at least 5 points, up to max: 0.1 for each set of measurements with range at least half of full decay range (data below), up to max:	0.8 0.8	
0.9	0.3 for each of the 3 $\mu$ values not from part a (data below)	0.9	
0.7	Graph plotted correctly: Axes, title labelled correctly Points plotted correctly 0.1 each Fit confirms linear hypothesis	0.1 0.4 0.2	
<b>Task 5: 3.0</b>	Use of intercept to get $k_0^2$ Use of $\lambda_0 = 2\pi/k_0$ Value for $29 \text{ cm} \leq \lambda_0 \leq 39 \text{ cm}$ <i>OR</i> Value for $24 \text{ cm} \leq \lambda_0 \leq 44 \text{ cm}$ Value for $n = \lambda_0/\lambda = 7.3 \pm 0.5$ <i>OR</i> Value for $n$ consistent with calculations but outside range	0.5 0.3 1.0 (0.5) 1.2 (0.5)	

**Data:**

$D$ (mm)	$\mu$ ( $\text{m}^{-1}$ )	Full decay range (mm)
41	109.4	90
46	94.6	105
59	73.1	140
100	42.6	260